Research Paper
Impact Factor: 4.164
Peer Reviewed Monthly Journal
www.ijmdrr.com

IJMDRR
E- ISSN –2395-1885
ISSN -2395-1877

SOME STUDIES ON DOMINATION NUMBER, BONDAGE NUMBER AND AVERAGE DISTANCE OF CIRCULAR-ARC GRAPHS

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Abstract
This network consists of communication links all distance between affixed set of sites. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. The problem is to select the smallest set of sites at which transmitters are placed so that every site in the network that does not have a transmitters is joined by a direct communication link to the site, which has a transmitter then this problem reduces to that of finding a minimum dominating set in the graph corresponding to this network. Suppose communication network does not work due to link failure. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. This leads to the introducing of the concept of the bondage number and the average distance. In this paper is some studies on the bondage number and the average distance of circular - arc graph.

Key Words: Combinatorial, Structures, Communication, Network, Transmitter, Average.

Introduction
A graph G=(V,E) is called acircular-arc graph or an intersecting graph for a finite family A of a non empty set if there is a one to one correspondence between A and B . Such that two sets in A have non empty intersection if and only if there corresponding vertices in V are adjacent to each other. We call A an intersection model of G for an intersection model A we use G(A) to denote the intersection graph for G. If A is a family of arcs on a circle, then G is called circular-arc graph for A and A is called a circular-arc model of G[2]. Circular-arc graphs have many applications in different fields such as genetic research, traffic control, computer complierdesign etc.

Let A= $\{A_1, A_2, A_3,\ldots\ldots A_n\}$ be a family of n arcs on a circle C. Each end point of the arcs in assigned a positive integer called a coordinate. The end point of each arc are located on the circumference are C in the ascending orderof the values of the coordinates in the clock wise direction. For convenience, each arcs are A_i, i=1,2,3…..n is represented as (h_i,t_i). Where h_i is the head point and t_i is the tail point respectively that starting and ending point$_s$ of the arc when it is traversed in counter clock wise manner, starting with an arbitrary chosen point on C which is not an end of any arc in A.Without loss of generality,

A subset D of v is said to be a dominating set of G if every vertex not in D is adjacent to vertex in D.The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G[1]. The bondage number b(G) of a non empty graph G is the minimum cardinality[5,6] among all sets of edges E_1 for which $\gamma(G-E_1)> \gamma(G)$.

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In this section we discuss about the computation of average distance of circular arc graph G. The average distance μ(G) of a connected circular arc graph is defined to be the average of all distances in G[3,4].

$$\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{A_i,A_j\in v(G)\\ A_i\neq A_j}}\delta(A_i,A_j).$$

Where δ(x,y) denotes the length of shortest path joining the vertices x and y . The average distance can be used as a tool in analytic networks where the performance time is proportions to the distance between any two nodes. It is a measure of the time needed in the average case as opposed to the diameter, which indicates the maximum performance time. And also the formulated of the walk, length of a walk, eccentricity, radius and diameter of the graph.

Theorem

Let A={A_1,A_2,……..A_n} be a circular arc family and let G be a circular arc graph corresponding to circular arc family without isolatedvertices of G. Then the domination number $\gamma(G)$, the average distance $\mu(G)$ and the bondage number b(G) is

$\mu(G) \geq \gamma(G) \geq$ b(G)

Proof

We consider the circular arc graph corresponding to circular arc family, In this it will arise three cases.

i. the domination number γ(G)

ii. The bondage number b(G) = γ(G-e) >γ(G)

iii. The average distance of G, $\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{A_i,A_j\in v(G)\\ A_i\neq A_j}}\delta(A_i,A_j).$

Let A={A_1,A_2,……..A_n} be a circular arc family and let G be a circular arc graph corresponding to circular arc family A. Let (A_i,A_j) be any two arcs in A, which satisfy the hypothesis of the theorem. Then clearly $A_i \in \gamma(G)$. Where γ(G) is a minimal domination number of a circular arc graph G because there is no other arc in A other than A_i that dominates A_j. Consider the edge e = (A_i,A_j) in G. If we remove this edge from a circular arc graph G. Then $A_j$becomes an isolated in circular arc graph G-e, as there is no other vertex in G other than A_i. That is adjacent with A_j.

Hence $\gamma_1(G) = \gamma(G) \cup (A_j)$ becomes a domination number of a circular arc graph G-e and since γ(G) is a minimum dominating set it follows that $\gamma_1(G)$ is also a minimum domination number of circular arc graph G-e.

Therefore we get $|\gamma_1(G)| = \gamma(G\text{-}e) = |\gamma(G)| +1 > |\gamma_1(G)|$

Thus the bondage number b(G) =1. As follows circular-arc graph G.

Now we will find the average distance μ(G) of a circular arc graph G, where μ(G) is a average distance of G, n is the number of arcs and δ (A_i,A_j) is the distance of the circular arc G. In this fact first we will discuss the distance of circular arc graph G for any two vertices (A_i,A_j) in a circular arc graph G, the distance from A_i to A_j is denoted by δ (A_i,A_j) and defined as the length of a shortest A_i - $A_j$path in circular-arc graph G.

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In this section we discuss about the computation of average distance of a circular-arc graph G. The average distance μ(G) of a connected a circular arc graph is defined to be the average of distance in G.Where $\mu(G) = \frac{1}{2\ {}^nc_2} \sum_{\substack{A_i, A_j \in v(G) \\ A_i \neq A_j}} \delta(A_i, A_j)$.

Where $\mu(G) = 2.118$ from the circular-arc graph G.
Thus the theorem is hold.

Practical procedure for theorem .1

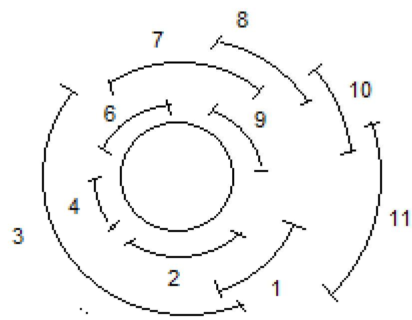

Fig. 1: Circular-arc family A

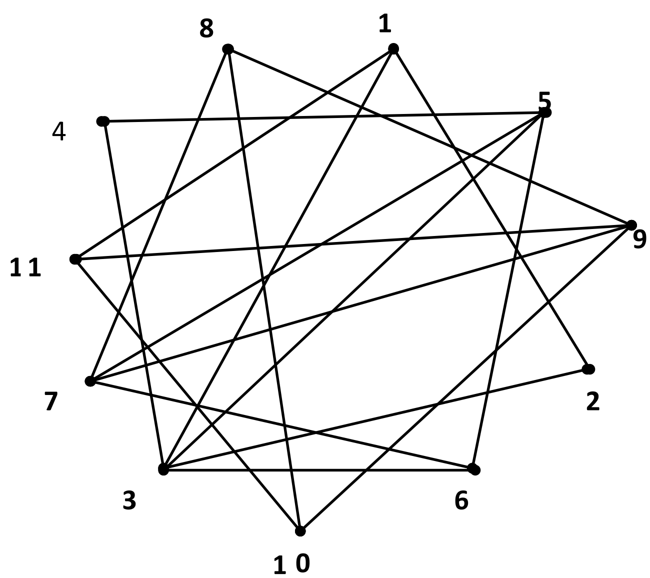

Fig. 2: Circular-arc graph G

Dominating Set = {3,9}, γ(G)=2
Remove the one edge from graph G
i.e G-e = G^1= G-(7,9), where e = (7,9)
Then the graph is

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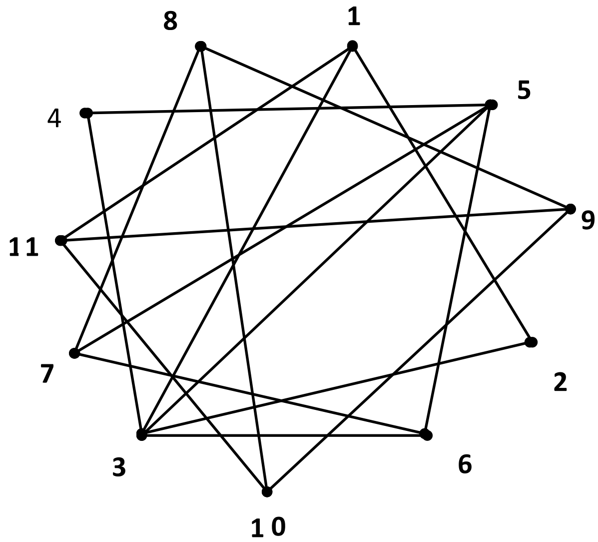

Fig. 3: Circular-arc graph (G-e)

Then the dominating set of a graph G-e $=G^1=\{3,7,10\}$
γ (G-e) = 3
γ(G-e) > γ(G)
Therefore the bondage number b(G) = 1
Therefore $\gamma(G) \geq b(G)$

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To Find the Distances from G

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=2	d(5,1)=2	d(6,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=2	d(5,2)=2	d(6,2)=2
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=1	d(6,3)=1
d(1,4)=2	d(2,4)=2	d(3,4)=1	d(4,4)=0	d(5,4)=2	d(6,4)=2
d(1,5)=2	d(2,5)=2	d(3,5)=1	d(4,5)=2	d(5,5)=0	d(6,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=1	d(4,6)=2	d(5,6)=1	d(6,6)=0
d(1,7)=3	d(2,7)=3	d(3,7)=2			

d(7,1)=3	d(8,1)=3	d(9,1)=	d(10,1)=2	d(11,1)=1
d(7,2)=3	d(8,2)=4	d(9,2)=	d(10,2)=3	d(11,2)=2
d(7,3)=2	d(8,3)=3	d(9,3)=	d(10,3)=3	d(11,3)=2
d(7,4)=3	d(8,4)=4	d(9,4)=	d(10,4)=4	d(11,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=	d(10,5)=3	d(11,5)=3
d(7,6)=1	d(8,6)=3	d(9,6)=2	d(10,6)=3	d(11,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2	d(11,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=2
d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=1
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1
d(7,11)=3	d(8,11)=2	d(9,11)=2	d(10,11)=1	d(11,11)=0

TO FIND THE AVERAGE DISTANCE OF G

Table 1

vertices	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	2	2	2	3	3	2	2	1
2	1	0	1	2	2	2	3	4	3	3	2
3	1	1	0	1	1	1	2	3	3	3	2
4	2	2	1	0	2	2	3	4	4	4	3
5	2	2	1	2	0	1	1	2	2	3	3
6	2	2	1	2	1	0	1	2	2	3	3
7	3	3	2	3	1	1	0	1	1	2	3
8	3	4	3	4	2	3	1	0	1	1	2
9	2	3	3	4	2	2	1	1	0	1	2
10	2	3	3	4	3	3	2	1	1	0	1
11	1	2	2	3	3	3	2	2	1	1	0
Total	19	23	18	27	19	20	19	23	20	23	22

Therefore Average distance $\mu(G) = \frac{1}{2\ {}^{n}c_2} \sum_{\substack{a_i, a_j \in v(G) \\ a_i \neq a_j}} \delta(a_i, a_j).$

$$\mu(G) = \frac{1}{11 \times 10} \times 233$$

$$\mu(G) = 2.118$$

$\therefore$ $\mu(G) \geq b(G)$

Therefore $\mu(G) \geq \gamma(G) \geq b(G)$.

In this graph we have $2.118 \geq 2 \geq 1$.

The comparison of the domination number, average distance and bondage number of G^1=G-e

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